ECON UN3265 ▪ MONEY AND BANKING ▪ SUMMER 2026 ▪ SESSION 2

# CHAPTER 4
# The Meaning of Interest Rates

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# Outline

# Reading and objectives

- Mishkin, *The Economics of Money, Banking, and Financial Markets*, 13th ed.
- This deck: **Chapter 4** — The Meaning of Interest Rates.
- Follows the Chapter 3 deck for this session.

## Learning objectives

**4.1** Calculate the present value of future cash flows and the yield to maturity on the four types of credit market instruments.

**4.2** Distinguish among yield to maturity, current yield, rate of return, and rate of capital gain.

**4.3** Interpret the distinction between real and nominal interest rates.

PART 1

# Measuring Interest Rates: Present Value

# A dollar today is worth more than a dollar tomorrow

## Present value

A dollar paid to you one year from now is *less valuable* than a dollar paid to you today.

- Why? A dollar you have today can be deposited to earn interest — so it grows to more than a dollar by next year.
- **Motivating question:** a $20 million lottery prize paid as $1 million per year for 20 years — is that really worth $20 million today?
- No. Later payments are worth less today. We need a way to compare cash flows at different dates.

## The present value formula

If the interest rate is $i$, then \$1 today grows to $\$(1+i)$ in one year, $\$(1+i)^2$ in two years, and $\$(1+i)^n$ in $n$ years. Running this in reverse gives the **present value** of a future cash flow:

### Simple present value

$$PV = \frac{CF}{(1+i)^n}$$

$PV =$ today's value   $CF =$ future cash flow   $i =$ interest rate   $n =$ years from now.

A higher interest rate, or a more distant payment, means a *lower* present value. Payments at different dates cannot be compared directly — discount them to a common date first.

# How much is that jackpot worth?

- You win the \$20 million New York State Lottery — \$1 million a year for 20 years.
- Each payment must be discounted to today. At an interest rate of $i = 10\%$:

$$PV = \frac{\$1\text{m}}{1.10} + \frac{\$1\text{m}}{(1.10)^2} + \cdots + \frac{\$1\text{m}}{(1.10)^{20}}$$

- The sum is far below \$20 million — roughly \$8.5 million in today's dollars.

**Takeaway**

To claim the winner has won \$20 million ignores *discounting the future*. Present value is the tool for valuing any stream of payments.

PART 2

# Four Credit Market Instruments

# Four types of credit market instrument

Debt instruments differ in the *timing* of their cash flows:

1. **Simple loan** — the borrower receives funds and repays the principal plus an interest payment at the maturity date.
2. **Fixed-payment loan** — the borrower makes the *same payment every period* (principal + interest blended) until maturity. Also called a *fully amortized loan* — e.g. mortgages, car loans.
3. **Coupon bond** — pays a fixed *coupon* payment each year until maturity, then repays the *face (par) value*. E.g. Treasury notes/bonds, corporate bonds.
4. **Discount bond** (zero-coupon bond) — bought at a price *below* face value; pays no coupons; repays the face value at maturity. E.g. Treasury bills.

PART 3

# Yield to Maturity

# Yield to maturity

**Yield to maturity (YTM)**

The interest rate that equates the present value of the cash flows from a debt instrument with its value (price) today.

- It is the single $i$ that makes "price $=$ present value of all future payments" hold.
- Economists consider YTM the **most accurate measure of interest rates**. When we say "the interest rate," we mean the yield to maturity.

# YTM on a simple loan and a fixed-payment loan

**Simple loan** — borrow \$100, repay \$110 in one year:

$$\$100 = \frac{\$110}{1+i} \implies i = 0.10 = 10\%.$$

For a simple loan, the simple interest rate *equals* the yield to maturity. **Fixed-payment loan** — loan value $LV$, fixed yearly payment $FP$, $n$ years:

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}.$$

Solve for $i$: the YTM is the rate that makes the present value of all the equal payments add up to the amount borrowed.

# YTM on a coupon bond

A coupon bond pays coupon $C$ each year for $n$ years and repays face value $F$ at maturity. Its price $P$ is the present value of that stream:

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}.$$

Three facts fall out of this equation:

- When the bond is priced *at* its face value, YTM $=$ the coupon rate.
- The price of a coupon bond and its YTM are **negatively related**.
- When the price is *below* face value, YTM $>$ coupon rate (and vice versa).

# Table 1 Price and Yield Move in Opposite Directions

| Price of bond ($) | Yield to maturity (%) |
|---:|---:|
| 1,200 | 7.13 |
| 1,100 | 8.48 |
| 1,000 | 10.00 |
| 900 | 11.75 |
| 800 | 13.81 |

10%-coupon-rate bond, $1,000 face value, maturing in ten years.

*Mishkin Ch. 4, Table 1. At a price of $1,000 the yield (10%) equals the coupon rate.*

## Consols and discount bonds

**Consol (perpetuity)** — a bond with no maturity date that pays a fixed coupon $C$ *forever*. Its price collapses to a simple ratio:

$$P_c = \frac{C}{i} \qquad \Longleftrightarrow \qquad i = \frac{C}{P_c}.$$

This ratio, $C/P$, is the **current yield** — an easy approximation to the YTM of a long-term coupon bond. **Discount bond** — bought at price $P$ below face value $F$, no coupons:

$$i = \frac{F - P}{P}.$$

The YTM is the increase in value over the year as a fraction of the price — again *negatively related* to the current price.

PART 4

# Interest Rates and Returns

# The rate of return

The interest rate is *not* the same as how well you did by holding a bond. That is the **rate of return**.

**Rate of return**

$$R = \frac{C + (P_{t+1} - P_t)}{P_t} = \underbrace{\frac{C}{P_t}}_{\text{current yield } i_c} + \underbrace{\frac{P_{t+1} - P_t}{P_t}}_{\text{rate of capital gain } g}$$

- The return = current yield + rate of capital gain.
- The return equals the YTM *only if* the holding period equals the time to maturity.

# A rise in rates can mean a capital loss

- A *rise* in interest rates lowers bond prices — a capital *loss* for anyone holding a bond whose maturity is longer than their holding period.
- The more *distant* a bond's maturity, the larger the price swing for a given change in interest rates.
- So even a bond with a high coupon can deliver a **negative return** if interest rates rise while you hold it.

**Interest rates need not be positive**

Recent experience in Japan and several European economies shows interest rates can even go *negative*.

# Table 2 One-Year Returns When Rates Rise from 10% to 20%

| Years to maturity | Initial current yield (%) | Initial price ($) | Price next year ($) | Rate of capital gain (%) | Rate of return (%) |
|---|---|---|---|---|---|
| 30 | 10 | 1,000 | 503 | $-49.7$ | $-39.7$ |
| 20 | 10 | 1,000 | 516 | $-48.4$ | $-38.4$ |
| 10 | 10 | 1,000 | 597 | $-40.3$ | $-30.3$ |
| 5 | 10 | 1,000 | 741 | $-25.9$ | $-15.9$ |
| 2 | 10 | 1,000 | 917 | $-8.3$ | $+1.7$ |
| 1 | 10 | 1,000 | 1,000 | $0.0$ | $+10.0$ |

*Mishkin Ch. 4, Table 2. All bonds carry a 10% coupon rate. Longer maturity ⇒ larger loss.*

# Interest-rate risk

**Interest-rate risk**

The riskiness of an asset's return that arises from changes in interest rates.

- Prices and returns for **long-term bonds** are more *volatile* than those for short-term bonds.
- There is **no interest-rate risk** for any bond whose time to maturity exactly matches the holding period — you collect the promised yield regardless of price swings in between.

PART 5

# Real and Nominal Interest Rates

# Nominal vs. real interest rates

- The **nominal interest rate** makes no allowance for inflation — it is the rate quoted on the loan.
- The **real interest rate** is adjusted for changes in the price level, so it more accurately reflects the true cost of borrowing and the true reward to lending.
- Two versions:
  - *Ex ante* real rate — adjusted for *expected* inflation (relevant for decisions).
  - *Ex post* real rate — adjusted for *actual* inflation (known only afterward).

# The Fisher equation

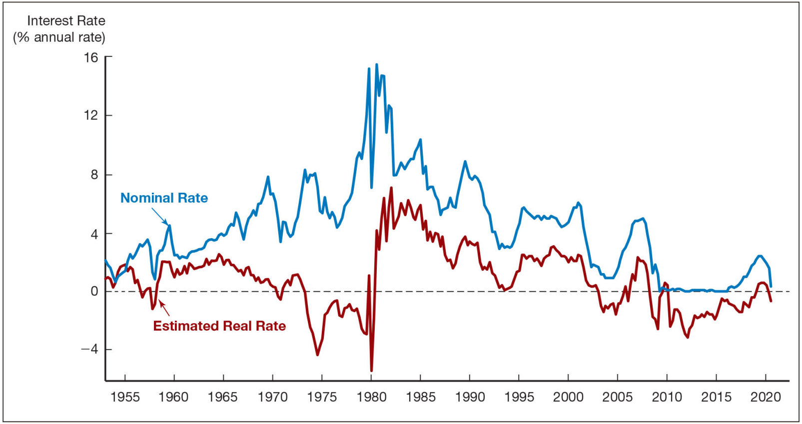
**Fisher equation**

$$i = r + \pi^e \qquad \Longleftrightarrow \qquad r = i - \pi^e$$

$i$ = nominal interest rate   $r$ = real interest rate   $\pi^e$ = expected inflation.

- When the *real* rate is low, there are greater incentives to borrow and fewer to lend.
- Example: a 14% mortgage with 15% expected inflation has a real rate of $-1\%$ — borrowing is cheap in real terms despite the high nominal rate.
- The real rate is the better indicator of the incentives facing borrowers and lenders.

# Figure 1 Real and Nominal Interest Rates (3-Month T-Bill), 1953–2020




*Mishkin Ch. 4, Figure 1. Source: Federal Reserve Bank of St. Louis (FRED); real rate estimated.*

PART 6

# Wrap-up

# Key terms from Chapter 4

- present value, discounting
- simple loan, fixed-payment loan
- coupon bond, discount bond
- face (par) value, coupon rate
- yield to maturity
- current yield, consol / perpetuity

- rate of return
- rate of capital gain
- interest-rate risk
- nominal vs. real interest rate
- ex ante / ex post real rate
- Fisher equation

## Looking ahead

- **Session 3:** The Behavior of Interest Rates; Risk and Term Structure (Mishkin Ch. 5–6).
- *Quiz 1* at the start of Session 3 — covers Week 1, Chapters 1–4.
- The present-value and yield-to-maturity tools from this chapter are used throughout the rest of the course.